Objectives: (things you should know and/or be able to do)

- Be familiar with and be able to derive the single-phase, pseudosteady-state flow relations for compressible liquids in a radial flow system. In particular, you should be able to derive the following:
  - \( p_r - p_{wf} \) flow relation:
    \[
    p_r = p_{wf} + \frac{1}{c_r} \frac{qB\mu}{kh} \left[ \frac{r_e^2}{(r_e^2-r_w^2)} \ln \left( \frac{r}{r_w} \right) - \frac{1}{2} \frac{(r^2-r_w^2)}{(r_e^2-r_w^2)} + s \right]
    \]
  - \( \bar{p} - p_{wf} \) flow relation:
    \[
    \bar{p} = p_{wf} + \frac{1}{c_r} \frac{qB\mu}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right] \quad \text{(Well Centered in a Circular Reservoir)}
    \]
    \[
    \bar{p} = p_{wf} + \frac{1}{c_r} \frac{qB\mu}{kh} \left[ \frac{1}{2} \ln \left( \frac{4A}{e\gamma r_w^2 C_A} \right) + s \right] \quad \text{(General Formulation)}
    \]

where:

\( \gamma = 0.577216 \ldots \) (Euler's Constant)

\( C_A = \) Dietz "shape factor" (e.g., \( C_A = 31.62 \) for a well in a circular reservoir)
Objectives: (things you should know and/or be able to do)

- Be familiar with and be able to derive the single-phase, pseudosteady-state flow relations for compressible liquids in a radial flow system. In particular, you should be able to derive the following:

  - \( p(r, t) \) solution for pseudosteady-state flow conditions:
    \[
    p_r = p_i - \frac{qB\mu}{2\pi kh} \left[ \ln \left( \frac{r_e}{r} \right) + \frac{1}{2} \left( \frac{r^2-r_w^2}{r_e^2-r_w^2} \right) - \frac{3}{4} \right] - \frac{qB}{V_pc} t
    \]
    (Darcy Units)

    \[
    p_r = p_i - 141.2 \frac{qB\mu}{kh} \left[ \ln \left( \frac{r_e}{r} \right) + \frac{1}{2} \left( \frac{r^2-r_w^2}{r_e^2-r_w^2} \right) - \frac{3}{4} \right] - 5.615 \frac{qB}{V_pc} t
    \]
    (Field Units)

    where for field units we use \( t \) in days, and \( V_p \) in ft\(^3\).

  - \( p(r_w, t) \) solution for pseudosteady-state flow conditions:
    \[
    p_{wf} = p_i - 141.2 \frac{qB\mu}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right] - 5.615 \frac{qB}{V_pc} t
    \]
    (Field Units)
Pseudosteady-State Flow Relations for a Radial System
from Department of Petroleum Engineering Course Notes (1997)
Pseudosteady-State Flow in a Radial System

Physical Considerations

The physical concept of pseudosteady-state is defined as the condition where the pressure at all points in the reservoir changes at the same rate. Mathematically, this condition is given by:

\[ \frac{d}{dt} [p(r, t)]_r = \text{constant} \] (1)

Physically, this condition is illustrated by

\[ q = \text{constant} \]

where,

- \( p_i \) = wellbore pressure at time, \( t_i \)
- \( \bar{p}_i \) = average reservoir pressure at time, \( t_i \)
- \( p_e \) = external boundary pressure at time, \( t_i \)

Objectives: (for pseudosteady-state flow conditions)

1. Derive a pressure change relation (i.e., \( \frac{dp}{dt} \)) using the material balance relation.
2. Derive a relation between the average reservoir pressure, \( \bar{p}_i \) and the wellbore flowing pressure, \( p_f \).
3. Derive a pressure, radius, time (i.e., \( p(r, t) \)) solution of the radial flow diffusivity equation.

Material Balance Considerations

Recalling the material balance relation for a slightly compressible liquid, we have

\[ \bar{p} = p_i - \frac{B}{N_b} \frac{d}{dt} \] (2)

or, noting that \( N_b = V_p \) we obtain

\[ \bar{p} = p_i - \frac{B}{V_p c_t} \] (3)

For a cylindrical reservoir, we have

\[ N_p = \phi \pi (r_e^2 - r_w^2) \] (4)

Substituting Eq. 4 into Eq. 3 gives us

\[ \bar{p} = p_i - \frac{B}{\phi \pi (r_e^2 - r_w^2) c_t} \] (5)

Recalling the definition of the cumulative production, \( N_p \), we have

\[ N_p = \int_0^t q(t) \, dt \] (6)

Therefore,

\[ \frac{dN_p}{dt} = q \] (7)

Taking the derivative of Eq. 5 with respect to time

\[ \frac{dp}{dt} = - \frac{B}{\phi \pi (r_e^2 - r_w^2) c_t} q \] (8)

Note: all derivations are in "bar-" units unless otherwise noted.
**Pseudosteady-State Flow Solutions for the Radial Flow**

**Diffusivity Equation**

The governing partial differential equation for flow in porous media is called the "diffusivity" equation. The diffusivity equation for a slightly compressible liquid is given (without derivation) as

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{dp}{dr} \right] = \frac{q_{av}}{k} \frac{dp}{dt}
\]  

(9)

The significant assumptions made in Eq. 9 are:
- Slightly compressible liquid (constant compressibility)
- Constant fluid viscosity
- Single-phase liquid flow
- Gravity and capillary pressure are neglected
- Constant permeability
- Horizontal radial flow (no vertical flow)

If we assume that the flowrate, \( q \), is constant then \( \frac{dp}{dt} \) is also constant, hence Eq. 10 is constant as well. Assuming \( q \) is constant, then

\[
\frac{dp}{dt} = \frac{dp}{dt} = -\frac{B}{\varphi h \pi (r^2 - r_0^2) c_k} \quad q = \text{constant}
\]

(10)

Substituting Eq. 10 into Eq. 9 (we note that partial derivatives are now expressed as ordinary derivatives), this gives

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{dp}{dr} \right] = \frac{\frac{d}{dt} \left[ \frac{q_{av}}{k} \right]}{\frac{\varphi h \pi (r^2 - r_0^2) c_k}} \quad q = \text{constant}
\]

or, reducing

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{dp}{dr} \right] = -\frac{q_{av}}{\pi k h (r^2 - r_0^2)}
\]

(11)

**Defining**

\[
c = \frac{q_{av}}{\pi k h (r^2 - r_0^2)}
\]

(12)

Substituting Eq. 12 into Eq. 11 we have

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{dp}{dr} \right] = -c
\]

Separating

\[
d \left[ r \frac{dp}{dr} \right] = -cr dr
\]

Integrating (indefinite integration)

\[
\int d \left[ r \frac{dp}{dr} \right] = -c \int r dr
\]

Completing

\[
r \frac{dp}{dr} = -c E^2 + c_1
\]

(14)

Multiplying through Eq. 14 by \( \frac{1}{r} \) gives us

\[
\frac{dp}{dr} = -\frac{c}{z} E^2 + c_1 \frac{1}{r}
\]

(15)

For pseudosteady-state we assume a closed reservoir, that is

\[
\left[ \frac{dp}{dr} \right]_{re} = 0
\]

or

\[
\left[ \frac{dp}{dr} \right]_{re} = 0 = -\frac{c}{z} E^2 + c_1 \frac{1}{re}
\]

Solving for \( c_1 \) gives

\[
c_1 = \frac{c}{z} E^2
\]

(16)
Substituting Eq. 16 into Eq. 15 gives
\[ \frac{dp}{dr} = \frac{c}{2} \left[ \frac{r e^2}{r} - r \right] \]  \hspace{1cm} (17)

Multiplying through Eq. 17 by \( dr \) gives us
\[ dp = \frac{c}{2} \left[ \frac{r e^2}{r} - r \right] dr \]

Integrating across the reservoir, we have
\[ \int_{p_{rf}}^{p_r} dp = \frac{c}{2} \int_{r_{fw}}^{r} \left[ \frac{r e^2}{r} - r \right] dr \]  \hspace{1cm} (18)

Completing the integration
\[ p_r - p_{rf} = \frac{c}{2} \left[ r e^2 \ln(r) \bigg|_{r_{fw}}^{r} - \frac{r^2}{2} \bigg|_{r_{fw}}^{r} \right] \]

or
\[ p_r - p_{rf} = \frac{c}{2} \left[ r e^2 \ln\left(\frac{r}{r_{fw}}\right) - \frac{1}{2} (r^2 - r_{fw}^2) \right] \]  \hspace{1cm} (19)

Recalling Eq. 12
\[ c = \frac{98 \mu}{\pi k h (r^2 - r_{fw}^2)} \]  \hspace{1cm} (20)

Substituting Eq. 20 into Eq. 19, we obtain
\[ p_r - p_{rf} = \frac{98 \mu}{2 \pi k h (r^2 - r_{fw}^2)} \left[ r e^2 \ln\left(\frac{r}{r_{fw}}\right) - \frac{1}{2} (r^2 - r_{fw}^2) \right] \]

Expanding through with the \( \frac{1}{(r^2 - r_{fw}^2)} \) term gives
\[ p_r - p_{rf} = \frac{98 \mu}{2 \pi k h} \left[ \frac{r e^2}{r_{fw}} \ln\left(\frac{r}{r_{fw}}\right) - \frac{1}{2} \left(\frac{r^2 - r_{fw}^2}{(r^2 - r_{fw}^2)}\right) \right] \]  \hspace{1cm} (21)

Eq. 21 is our final result (in "Darcy" units).
Substituting Eq. 26 into Eq. 25 gives

\[ \bar{P}_r = \frac{2}{(r^2 - r_w^2)} \left[ \frac{r_w^2}{2(\chi r_w^2)} \ln \left( \frac{r}{r_w} \right) - \frac{1}{2} \left( \frac{r^2 - r_w^2}{r^2 - r_w^2} \right) \right] r \, dr \]  

(27)

Separating

\[ \bar{P}_r = \frac{2}{(r^2 - r_w^2)} \left[ \frac{r_w^2}{2(\chi r_w^2)} \int_{r_w}^{r} r \, dr \right] \]

\[ + \frac{2}{(r^2 - r_w^2)} \frac{g \mu}{2 \pi k h} \frac{r_w^2}{2(\chi r_w^2)} \int_{r_w}^{r} r \ln \left( \frac{r}{r_w} \right) \, dr \]

\[ - \frac{2}{(r^2 - r_w^2)} \frac{g \mu}{2 \pi k h} \frac{1}{2(\chi r_w^2)} \int_{r_w}^{r} r^3 \, dr \]

\[ + \frac{2}{(r^2 - r_w^2)} \frac{g \mu}{2 \pi k h} \frac{r_w^2}{2(\chi r_w^2)} \int_{r_w}^{r} r \, dr \]  

(28)

Isolating terms and evaluating each integral, we have

\[ \int_{r_w}^{r} r \, dr = \frac{1}{2} (r^2 - r_w^2) \]  

(29)

\[ \int_{r_w}^{r} r^3 \, dr = \frac{1}{4} (r^4 - r_w^4) \]  

(30)

\[ \int_{r_w}^{r} r \ln \left( \frac{r}{r_w} \right) \, dr = ? \]

Starting with the fundamental form of the logarithm integral, we have

\[ \int x \ln(x/c) \, dx \]

... integration by parts

\[ m = \ln(x/c) \]

\[ dv = x \, dx \]

\[ du = \frac{1}{x} \, dx \]

\[ v = \frac{1}{2} x^2 \]

Then

\[ \int x \ln(x/c) \, dx = \frac{1}{2} x^2 \ln(x/c) - \frac{1}{2} \int x \, dx \]

Reducing

\[ \int x \ln(x/c) \, dx = \frac{1}{2} x^2 \ln(x/c) - \frac{1}{4} x^2 \]

Therefore

\[ \int_{r_w}^{r} r \ln \left( \frac{r}{r_w} \right) \, dr = \left[ x \ln \left( \frac{r}{r_w} \right) - \frac{1}{4} r^2 \right]_{r_w}^{r} \]

\[ = \frac{1}{2} r^2 \ln \left( \frac{r}{r_w} \right) - \frac{1}{4} r^2 - \frac{1}{4} \left[ r_w^2 \ln \left( \frac{r_w}{r_w} \right) - \frac{1}{4} r_w^2 \right] \]

or finally, we have

\[ \int_{r_w}^{r} r \ln \left( \frac{r}{r_w} \right) \, dr = \frac{1}{2} r^2 \ln \left( \frac{r}{r_w} \right) - \frac{1}{4} (r^2 - r_w^2) \]  

(31)

Obviously, the integral of the logarithm term will require a little work to resolve, we could simply look up the appropriate result in a suitable text — but deriving the required result will be enlightening.
Derivation of the Pseudosteady-State Flow Relations for a Radial System

Substituting Eqs. 29-31 into Eq. 28 gives

\[
\bar{p}_r = \frac{z}{(r^2-r_w^2)} \frac{1}{Z} \left[ r^2 \ln \left( \frac{r}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

\[
+ \frac{z}{(r^2-r_w^2)} \frac{98m}{2\pi k} \left[ \frac{1}{Z} \left( \frac{r^2}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

\[
- \frac{z}{(r^2-r_w^2)} \frac{98m}{2\pi k} \left[ \frac{1}{Z} \left( \frac{r^2}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

Reducing,

\[
\bar{p}_r = \eta_{nf} + \frac{98m}{2\pi k} \frac{z}{(r^2-r_w^2)} \left[ \frac{1}{Z} \left( \frac{r^2}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

\[
- \frac{98m}{2\pi k} \frac{z}{(r^2-r_w^2)} \left[ \frac{1}{Z} \left( \frac{r^2}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

\[
+ \frac{98m}{2\pi k} \frac{z}{(r^2-r_w^2)} \frac{1}{Z} \frac{r^2}{r_w} \left( r^2-r_w^2 \right)
\]

Collecting,

\[
\bar{p}_r = \eta_{nf} + \frac{98m}{2\pi k} \frac{z}{(r^2-r_w^2)} \left[ \frac{1}{Z} \left( \frac{r^2}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

\[
- \frac{98m}{2\pi k} \frac{(r^2-r_w^2)}{4(\frac{r^2}{r_w})} + \frac{98m}{2\pi k} \frac{r^2}{Z} \left( r^2-r_w^2 \right)
\]

or "finally"

\[
\bar{p}_r = \eta_{nf} + \frac{98m}{2\pi k} \left[ \left( \frac{r^2}{r_w} \right) - \frac{1}{2} \right]
\]

\[
+ \frac{98m}{2\pi k} \frac{z}{(r^2-r_w^2)} \left[ \frac{1}{Z} \left( \frac{r^2}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

\[
- \frac{98m}{2\pi k} \frac{(r^2-r_w^2)}{4(\frac{r^2}{r_w})} + \frac{98m}{2\pi k} \frac{r^2}{Z} \left( r^2-r_w^2 \right)
\]

Eq. 32 (which is given in "Darcy units") is our fundamental linking relation between the wellbore and average reservoir pressures during pseudosteady-state flow. However, \(\bar{p}_r\) (the average reservoir pressure at a given radius, \(r\)) is of little use except as a rigorous linking relation for pressures in the reservoir.

In contrast, if we consider \(\bar{p}_r\) (i.e., \(p_r\) at \(r=r_w\)) we obtain the average reservoir pressure based on the entire reservoir volume. Such a result can be directly coupled with the material balance equation to develop a time-pressure relation for pseudosteady-state flow.

Evaluating Eq. 32 at \(r=r_w\) we have

\[
\bar{p}_r = \bar{p}_w = \eta_{nf}
\]

\[
+ \frac{98m}{2\pi k} \left[ \left( \frac{r^2}{r_w} \right) - \frac{1}{2} \right]
\]

\[
- \frac{98m}{2\pi k} \frac{(r^2-r_w^2)}{4(\frac{r^2}{r_w})} + \frac{98m}{2\pi k} \frac{r^2}{Z} \left( r^2-r_w^2 \right)
\]

\[
\text{Assuming that } r_w \gg r_w, \text{ then}
\]

\[
\frac{r^2}{(r^2-r_w^2)} \approx 1; \quad \frac{(r^2+r_w^2)}{(r^2-r_w^2)} \approx 1; \quad \frac{r^2}{r_w^2} \approx 0
\]

Substituting these expressions into Eq. 33, we obtain

\[
\bar{p}_r = \eta_{nf} + \frac{98m}{2\pi k} \left[ \left( \frac{r^2}{r_w} \right) - \frac{1}{2} \right]
\]

\[
+ \frac{98m}{2\pi k} \frac{z}{(r^2-r_w^2)} \left[ \frac{1}{Z} \left( \frac{r^2}{r_w} \right) - \frac{1}{4} (r^2-r_w^2) \right]
\]

or

\[
\bar{p}_r = \eta_{nf} + \frac{98m}{2\pi k} \left[ \left( \frac{r^2}{r_w} \right) - \frac{3}{4} \right]
\]

(Derivation of the Pseudosteady-State Flow Relations for a Radial System)
Summary of our results so far (using generalized units system)

Pressure at any radius:

\[ \bar{P}_r = P_{w} + \frac{98u}{c_r k_h} \left[ \frac{r_e^2}{(r_e^2 - r_w^2)} \ln \left( \frac{r}{r_w} \right) - \left( \frac{r^2 - r_w^2}{2} \right) \frac{1}{2} \right] - \frac{4}{3} \left( \frac{r^2 + r_w^2}{r_e^2 - r_w^2} \right) \]

Average Reservoir Pressure at any radius:

\[ \overline{\bar{P}} = \frac{\bar{P}_r}{k_h} \]

Average Reservoir Pressure at \( \bar{r}_e \) (Volumetric Average Pressure):

\[ \overline{\bar{P}} = \frac{\bar{P}_w}{k_h} + \frac{98u}{c_r k_h} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \right] \]

For a general reservoir geometry, Eq. 38 becomes

\[ \overline{\bar{P}} = \bar{P}_w + \frac{98u}{c_r k_h} \left[ \frac{1}{2} \ln \left( \frac{r_e^2}{r_w^2} \right) \right] \]

where

\[ \gamma = 0.577216 \quad \text{Euler's Constant} \]

\[ c_A = \text{Darcy "shape factor" (e.g., } c_A = 51.62 \text{ for circular reservoir)} \]

Table of units conversion factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Darcy Units</th>
<th>Field Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_r )</td>
<td>( \frac{2\pi}{1.127 \times 10^{-3}} ) or ( 7.081 \times 10^{-3} )</td>
<td>( 2\pi \times 8.527 \times 10^{-3} ) ( \text{or } 5.55 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

\[ \frac{r_e^2}{r_w^2} \ln \left( \frac{r}{r_w} \right) - \frac{4}{3} \frac{r_w^2}{r_e^2 - r_w^2} = \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \]

We now assume \( \frac{r_e^2}{(r_e^2 - r_w^2)} \approx 1 \) and \( \frac{r_w^2}{(r_e^2 - r_w^2)} \approx 0 \) gives

\[ \frac{r_e^2}{r_w^2} \ln \left( \frac{r}{r_w} \right) - \frac{4}{3} \frac{r_w^2}{r_e^2 - r_w^2} = \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \]
Assuming that \( r_e \gg r_w \) (i.e., \((r_e^2-r_w^2) \approx r_e^2\)) and rearranging, we have

\[
\ln \left[ \frac{r_e^2}{r_e^2} \right] - \frac{r_w^2}{r_e^2} + \frac{3}{4} = 0 \tag{39}
\]

Defining a dimensionless radius, \( \bar{r}_D \), we obtain

\[
\bar{r}_D = \frac{r_w}{r_e^2} \tag{40}
\]

Substituting Eq. 40 into Eq. 39 gives us

\[
\ln \left( \bar{r}_D \right) - \frac{1}{2} \bar{r}_D^2 + \frac{3}{4} = 0 \tag{41}
\]

Solving Eq. 41 for \( \bar{r}_D \), we obtain

\[
\bar{r}_D = 0.54928 \ldots \quad \text{(or \( \bar{r} = 0.54928 \pi r_e \))} \tag{42}
\]

Development of a \( \rho(r,t) \) Relation for Pseudosteady-State Flow

Our last objective is to develop a \( \rho(r,t) \) relation for pseudosteady-state flow in a bounded circular reservoir. Recalling the material balance relation (Eq. 5), we have

\[
\rho = \rho_i - \frac{q}{\phi \pi (r_e^2-r_w^2)} N_p \tag{5}
\]

For a constant flowrate, \( q \), we have

\[
N_p = \int_0^t q(t) \, dt = qt \tag{43}
\]

Substituting Eq. 43 into Eq. 5,

\[
\rho = \rho_i - \frac{48}{\phi \pi (r_e^2-r_w^2)} t \quad \text{(Darcy units)} \tag{44}
\]

Recalling the average reservoir pressure identity for a well centered in a bounded circular reservoir, we have

\[
\bar{p} = \rho_i f + \frac{98 \mu}{2 \pi kh} \left[ \ln \frac{r_e^2}{r_w^2} - \frac{3}{4} \right] \tag{45}
\]

Substituting Eq. 45 into Eq. 44 gives

\[
\rho_i f + \frac{98 \mu}{2 \pi kh} \left[ \ln \frac{r_e^2}{r_w^2} - \frac{3}{4} \right] = \rho_i - \frac{48}{\phi \pi (r_e^2-r_w^2)} t
\]

Rearranging

\[
\rho_i - \rho_i f = \frac{98 \mu}{2 \pi kh} \left[ \ln \frac{r_e^2}{r_w^2} - \frac{3}{4} \right] + \frac{48}{\phi \pi (r_e^2-r_w^2)} t \tag{46}
\]

or

\[
\rho_i - \rho_i f = \frac{98 \mu}{2 \pi kh} \left[ \ln \frac{r_e^2}{r_w^2} - \frac{3}{4} \right] + \frac{48}{\phi \pi (r_e^2-r_w^2)} t \tag{47}
\]

Recalling the wellbore-reservoir pressure relation (Eq. 26), we have (upon slight rearranging)

\[
\rho_i - \rho_i f = \frac{98 \mu}{2 \pi kh} \left[ \ln \frac{r_e^2}{r_w^2} - \frac{3}{4} \right] + \frac{1}{2} \left( \frac{r_e^2-r_w^2}{r_e^2-r_w^2} \right) \tag{48}
\]

Subtracting Eq. 48 from Eq. 47 and solving for \( \rho_i \) gives us

\[
\rho_i = \rho_i - \frac{98 \mu}{2 \pi kh} \left[ \ln \frac{r_e^2}{r_w^2} - \frac{3}{4} \right] - \frac{1}{2} \left( \frac{r_e^2-r_w^2}{r_e^2-r_w^2} \right)
\]

\[
- \frac{48}{\phi \pi (r_e^2-r_w^2)} t \tag{49}
\]

Assuming that \( r_e \gg r_w \) (i.e., \((r_e^2-r_w^2) \approx r_e^2\)) gives us

\[
\rho_i = \rho_i - \frac{98 \mu}{2 \pi kh} \left[ \ln \frac{r_e^2}{r_w^2} + \frac{1}{2} \left( \frac{r_e^2-r_w^2}{r_e^2-r_w^2} \right) - \frac{3}{4} \right] - \frac{48}{\phi \pi (r_e^2-r_w^2)} t \tag{50}
\]

(Derivation of the Pseudosteady-State Flow Relations for a Radial System)
Derivation of the Pseudosteady-State Flow Relations for a Radial System

In Field Units, we have

\[ \frac{P_r - P_w}{P_r - P_w} = \frac{98 \mu}{2 \pi kh} \left[ \ln \left( \frac{r_e}{r_w} \right) + \frac{1}{4} \left( \frac{r_e^2 - r_w^2}{r_e^2} \right) \ln \left( \frac{r_e}{r_w} \right) + \frac{1}{2} \left( \frac{r_e^2 - r_w^2}{r_e^2} \right) \right] - \frac{98}{V_P \mu} t \] (49)

and

\[ \frac{P_r - P_w}{P_r - P_w} = \frac{98 \mu}{2 \pi kh} \left[ \ln \left( \frac{r_e}{r} \right) + \frac{1}{4} \left( \frac{r_e^2 - r^2}{r_e^2} \right) \right] - \frac{98}{V_P \mu} t \] (50)

For \( t \) in hours we use \( 5.615/24 = 0.23395 \), \( V_P \) in \( ft^3 \).

Finally, for conditions at the well, we have

**Darcy Units:**

\[ \frac{P_w}{P_r} = \frac{98 \mu}{2 \pi kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \right] - \frac{98}{V_P \mu} t \] (53)

**Field Units:**

\[ \frac{P_w}{P_r} = \frac{98 \mu}{2 \pi kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \right] - 5.615 \frac{98}{V_P \mu} t \] (54)

\( (t \text{ in days}; V_P \text{ in } ft^3) \)

\[ \frac{P_w}{P_r} = \frac{98 \mu}{2 \pi kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \right] - 0.23395 \frac{98}{V_P \mu} t \] (55)

\( (t \text{ in hours}; V_P \text{ in } ft^3) \)

Recall that the pore volume, \( V_P \), is given by

\[ V_P = \phi \pi (r_e^2 - r_w^2) = \phi \pi \Delta r^2 \] (4)
Illustrations of Pseudosteady-State Performance in Radial Flow Systems

Figure 2 - Pressure Distribution during Constant Rate Transient Flow Drawdown
Figure 7 - Reservoir Pressure Distribution during Constant Wellbore Pressure Transient Flow Drawdown

(Blasingame, T.A.: Variable-Rate Analysis: Transient and Pseudosteady-State Methods of Interpretation and Application, M.S. Thesis, Texas A&M University (1986))
\[ p_r = p_i - 141.2 \frac{qB \mu}{kh} \left[ \ln \left( \frac{r_e}{r} \right) + \frac{1}{2} \frac{(r_e^2 - r_w^2)}{(r_e^2 - r_w^2)} - \frac{3}{4} \right] - 5.615 \frac{qB}{V_p c_t} t \]  
(Field Units)

Figure 52 - Reservoir Pressure Distribution During Constant Rate Post-Transient Flow Drawdown, Homogeneous Reservoirs
Figure 57 - Reservoir Pressure Distribution During Constant Wellbore Pressure Post-Transient Flow Drawdown, Homogeneous Reservoirs
Pressure Trends
from Department of Petroleum Engineering Course Notes (2012)
Pressure Distributions: Solutions

All relations given in FIELD units.

Steady-State Solution:

\[ p_r = p_w + 141.2 \frac{q_{sc} B \mu}{kh} \ln\left(\frac{r}{r_w}\right) \quad [p_r - p_{wf} \text{ form}] \]

\[ p_r = p_e - 141.2 \frac{q_{sc} B \mu}{kh} \ln\left(\frac{r_e}{r}\right) \quad [p_r - p_e \text{ form}] \]

Radius of Investigation:

Full Solution: \( q_{sc} = \text{constant} \)

\[ p_D = \frac{1}{141.2} \frac{kh}{q B \mu} (p_i - p_r) \]

\[ \approx \frac{1}{2} E_1 \left[ \frac{r_D^2}{4t_D} \right] - \frac{1}{2} E_1 \left[ \frac{r_eD}{4t_D} \right] + 2 \frac{t_D}{r_eD} \exp \left[ -\frac{r_eD}{4t_D} \right] + \left[ \frac{r_D^2}{2r_eD} - \frac{1}{4} \right] \exp \left[ -\frac{r_eD}{4t_D} \right] \]

(Variou Notes)
Pressure Distributions for Transient Radial Flow

- Note the effect of the drawdown.
- Note that the buildup pressure trends retrace last drawdown trend.
- Recall that all measurements are at the wellbore, we cannot "see" in the reservoir — our analyses are inferred from wellbore measurements.
The physical concept of the PSEUDOSTEADY-STATE FLOW condition is defined as the condition where the pressure at all points in the reservoir changes at the same rate. Mathematically, this condition is given by:

\[
\frac{d}{dt}[p(r,t)]_r = \text{constant}
\]
Pressure Distributions: Pseudosteady-State

Concept: (pressure changes at the same rate at all points in the reservoir)

\[
\frac{dp}{dr} = \text{constant}
\]

Reservoir Pressure Schematic:
Pseudosteady-State Flow: **Summary of Relations**

**\((p_r-p_{wf})\) Flow Relations: (Circular Reservoir)**

\[
p_r - p_{wf} = 141.2 \frac{qB\mu}{kh} \left[ \frac{r_e^2}{(r_e^2 - r_w^2)} \ln \left( \frac{r}{r_w} \right) - \frac{1}{2} \frac{(r^2 - r_w^2)}{(r_e^2 - r_w^2)} + s \right]
\]

**\((\bar{p}-p_{wf})\) Flow Relations: (\(\gamma = 0.577216\) Euler's constant)**

\[
\bar{p} = p_{wf} + 141.2 \frac{qB\mu}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right]
\]  
(Circular Reservoir)

\[
\bar{p} = p_{wf} + 141.2 \frac{qB\mu}{kh} \left[ \frac{1}{2} \ln \left( \frac{4}{e\gamma} \frac{A}{r_w^2} \frac{1}{CA} \right) + s \right]
\]  
(General Formulation)

**Time-Dependent Pseudosteady-State Flow Relations:**

\[
p_r = p_i - 141.2 \frac{qB\mu}{kh} \left[ \ln \left( \frac{r_e}{r} \right) + \frac{1}{2} \frac{(r^2 - r_w^2)}{2 (r_e^2 - r_w^2)} - \frac{3}{4} \right] - 5.615 \frac{qB}{V_p c_t} t
\]

\[
p_{wf} = p_i - 141.2 \frac{qB\mu}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right] - 5.615 \frac{qB}{V_p c_t} t
\]

(Various Notes)
Pseudosteady-State Flow: *Illustrative Behavior*

\[ r_{inv} = 2.434 \times 10^{-2} \sqrt[2]{\frac{k}{\phi \mu c_t}} t \]

Figure 2: Reservoir Pressure Distribution — Constant Rate Transient Flow Drawdown.
Pseudosteady-State Flow: *Illustrative Behavior*

Figure 7: Reservoir Pressure Distribution — Constant Wellbore Pressure Transient Flow Drawdown.

\[
\text{Radius, ft} \quad 10^{-1} \quad 1 \quad 10 \quad 10^2 \quad 10^3
\]

\[
\begin{align*}
\text{Reservoir Pressure Distribution} & \\
\text{during Constant Wellbore Pressure Transient Flow Drawdown} & \\
\left(1\right) r_1 &= 32.2 \text{ ft} \\
\left(2\right) r_2 &= 88.4 \text{ ft} \\
\left(3\right) r_3 &= 195 \text{ ft} \\
\left(4\right) r_4 &= 413 \text{ ft} \\
\end{align*}
\]

\[
\text{inv} \quad r_{inv} = 2.434 \times 10^{-2} \sqrt{\frac{k}{\phi \mu c_l}} t
\]
Pseudosteady-State Flow: *Illustrative Behavior*

\[ r_{inv} = 2.434 \times 10^{-2} \sqrt[2]{\frac{k}{\phi \mu c_t}} \]

**Figure 52:** Reservoir Pressure Distribution — Constant Rate Post-Transient Flow Drawdown, Homogeneous Reservoirs.
Pseudosteady-State Flow: *Illustrative Behavior*

**Figure 57: Reservoir Pressure Distribution — Constant Wellbore Pressure Post-Transient Flow Drawdown, Homogeneous Reservoirs.**

\[ r_{inv} = 2.434 \times 10^{-2} \sqrt{\frac{k}{\phi \mu}} \sqrt{t} \]

\[ t_1 = 1298 \text{ hr} \]
\[ t_2 = 5759 \text{ hr} \]
\[ t_3 = 12125 \text{ hr} \]
Reservoir Pressure Trends: **Questions to Consider**

**Q1. Why study "reservoir pressure trends?"**

**A1.** We can not measure pressure in the reservoir — only at the wellbore (or sandface). In order to estimate the behavior in the reservoir, we must use "model-based" pressure distributions.

**Q2. Isn't the use of a simple model too limiting?**

**A2.** Actually, no. Simple models are extremely consistent, and as such, even when "wrong," the "trend" behavior is typically quite repre-sentative.

**Q3. What is the "radius of investigation?"**

**A3.** For the infinite-acting radial flow case, the radius of investigation is the point in the reservoir where the logarithm of radius equation (straight line) intersects the initial reservoir pressure. It is a fictitious point, but it represents the "theoretical" location of the front of the pressure distribution front.

\[ r_{inv} = 2.434 \times 10^{-2} \sqrt{\frac{k}{\phi \mu c_t}} t \]